SPE 9723

Comparison of Solutions to a Three-Dimensional Black-Oil Reservoir Simulation Problem

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Summary

A comparison of solutions to a three-dimensional black-oil reservoir simulation problem is presented. The test of the problem and a brief description of the seven simulators used in the study are given.

Introduction

Seven companies participated in a reservoir simulation project to compare the results obtained by different black-oil simulators. The companies were chosen to give a good cross section of the solution methods used in the industry. The participants were Amoco Production Co., Computer Modelling Group of Calgary (CMG), Exxon Production Research Co., Intercomp Resource Development and Engineering Inc., Mobil Research and Development Corp., Shell Development Co., and Scientific Software Corp. (SSC). The paper presents the text of the problem, a comparison of results in graphical form, and a brief description of each model. The descriptions were supplied by the participants.

A variety of computers was used. Amoco used IBM 3033, IBM 370/168, and Amdahl V/6. CMG used Honeywell 6000 DPS, and Exxon used Amdahl 470/V5 and IBM 370/168. Intercomp used Cray-1 and Harris/7. Mobil and SSC used CDC Cyber 175, and Shell used Univac 1110/2C Level 36. The number of time steps and the central processor times varied considerably. Those interested in the actual values should contact the individual companies.

Except for Shell, all the participants used single-point upstream mobility weighting. Shell used two points upstream. Constraints and data are given in the text.

Statement of the Problem

Areal and cross-section views of the reservoir are given in Figs. 1 and 2. The grid system is given in Fig. 1. Stratification and reservoir properties are given in Fig. 2. The reservoir is initially undersaturated. A gas injection well is located at Grid Point (1, 1), and a producing well is located at Grid Point (10, 10). Pertinent data and constraints are given in Table 1. PVT properties and relative permeabilities are given in Tables 2 and 3. The participants were asked to make the runs and report the results described below.

Runs To Be Made

Case 1

Let the bubble-point (saturation) pressure be constant with a value equal to the original value.

Case 2

Let the saturation pressure vary with gas saturation – i.e., this is a variable saturation-pressure case. The PVT lines at pressures above the calculated saturation pressures are parallel to the original line.

Results To Be Reported

The following results are to be reported.

1. Plots of:
   a. Oil rate vs. time.
   b. GOR vs. time.
2. Report annually and at abandonment:
   a. The pressures of the cell where the injector and producer are located.*
   b. Gas saturation at Grid Points (1, 1, 1), (1, 1, 2), (1, 1, 3), (10, 1, 1), (10, 1, 2), (10, 1, 3), (10, 10, 1), (10, 10, 2), and (10, 10, 3).
3. Report at the end of 8 years:
   a. Tables of gas saturation.
   b. Tables of cell pressures.*
   c. Tables of saturation pressures for the variable saturations-pressure case.*

*Report of all pressures referred to a depth of 8,325 ft or at the center of the respective blocks. If both are available, report both.

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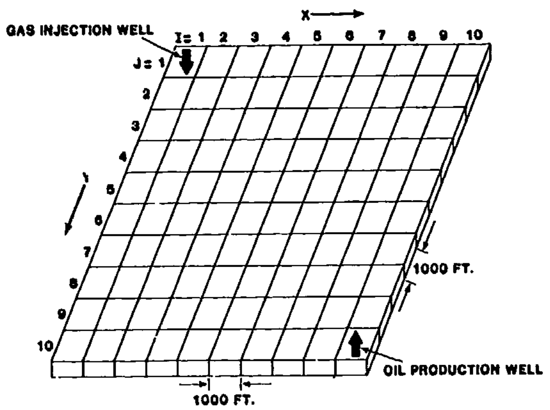

Fig. 1 – Reservoir and grid system.

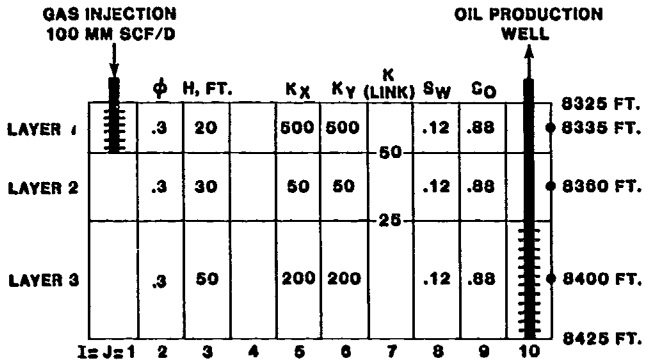

Fig. 2 – Diagonal cross section.

TABLE 1 – DATA AND CONSTRAINTS

Initial reservoir pressure, psia at 8,400 ft	4,800
Gas injection rate, MMscf/D	100
Maximum oil production rate, STB/D	20,000
Minimum oil rate, STB/D	1,000
Minimum flowing bottomhole pressure, psi	1,000
Maximum saturation change during time step	0.05
Rock compressibility, 1/psi	3×10^{-6}
Porosity value of 0.3 was measured at a base pressure of 14.7 psi	
Wellbore radius, ft	0.25
Skin	0
Capillary pressure	0
Reservoir temperature, &F	200
Gas specific gravity	0.792
Runs are terminated either at the end of 10 years or when GOR⅝20,000 scf/STB or when the oil production rate51,000 STB/D; whichever occurs first terminates the run.	

TABLE 2 – PVT PROPERTIES

Saturated Oil PVT Functions

Reservoir Pressure (psia)	FVF (RB/STB)	Viscosity (cp)	Density (lbm/cu ft)	Solution GOR (scf/stb)
14.7	1.0620	1.0400	46.244	1.0
264.7	1.1500	0.9750	43.544	90.5
514.7	1.2070	0.9100	42.287	180.0
1014.7	1.2950	0.8300	41.004	371.0
2014.7	1.4350	0.6950	38.995	636.0
2514.7	1.5000	0.6410	38.304	775.0
3014.7	1.5650	0.5940	37.781	930.0
4014.7	1.6950	0.5100	37.046	1270.0
5014.7	1.8270	0.4490	36.424	1618.0
9014.7	2.3570	0.2030	34.482	2984.0

Saturated Water PVT Functions

Reservoir Pressure (psia)	FVF (RB/bbl)	Viscosity (cp)	Density (lbm/cu ft)	Gas/Water Ratio (scf/bbl)
14.7	1.0410	0.3100	62.238	0.0
264.7	1.0403	0.3100	62.283	0.0
514.7	1.0395	0.3100	62.328	0.0
1014.7	1.0380	0.3100	62.418	0.0
2014.7	1.0350	0.3100	62.599	0.0
2514.7	1.0335	0.3100	62.690	0.0
3014.7	1.0320	0.3100	62.781	0.0
4014.7	1.0290	0.3100	62.964	0.0
5014.7	1.0258	0.3100	63.160	0.0
9014.7	1.0130	0.3100	63.959	0.0

Undersaturated Oil PVT Functions

Reservoir Pressure (psia)	FVF (RB/STB)	Viscosity (cp)	Density (lbm/cu ft)
4014.7	1.6950	0.5100	37.046
9014.7	1.5790	0.7400	39.768

Undersaturated Water PVT Functions

Reservoir Pressure (psia)	FVF (RB/bbl)	Viscosity (cp)	Density (lbm/cu ft)
4014.7	1.0290	0.3100	62.964
9014.7	1.0130	0.3100	63.959

Gas PVT Functions

Reservoir Pressure (psia)	FVF (RB/bbl)	Viscosity (cp)	Density (lbm/cu ft)	Pseudo Gas Potential $M(p)$ ($psia^2/cp$)
14.7	0.166666	0.008000	0.0647	0.
264.7	0.012093	0.009600	0.8916	0.777916 E+07
514.7	0.006274	0.011200	1.7185	0.267580 E+08
1014.7	0.003197	0.014000	3.3727	0.875262 E+08
2014.7	0.001614	0.018900	6.6806	0.270709 E+09
2514.7	0.001294	0.020800	8.3326	0.386910 E+09
3014.7	0.001080	0.022800	9.9837	0.516118 E+09
4014.7	0.000811	0.026800	13.2952	0.803963 E+09
5014.7	0.000649	0.030900	16.6139	0.112256 E+10
9014.7	0.000386	0.047000	27.9483	0.251845 E+10

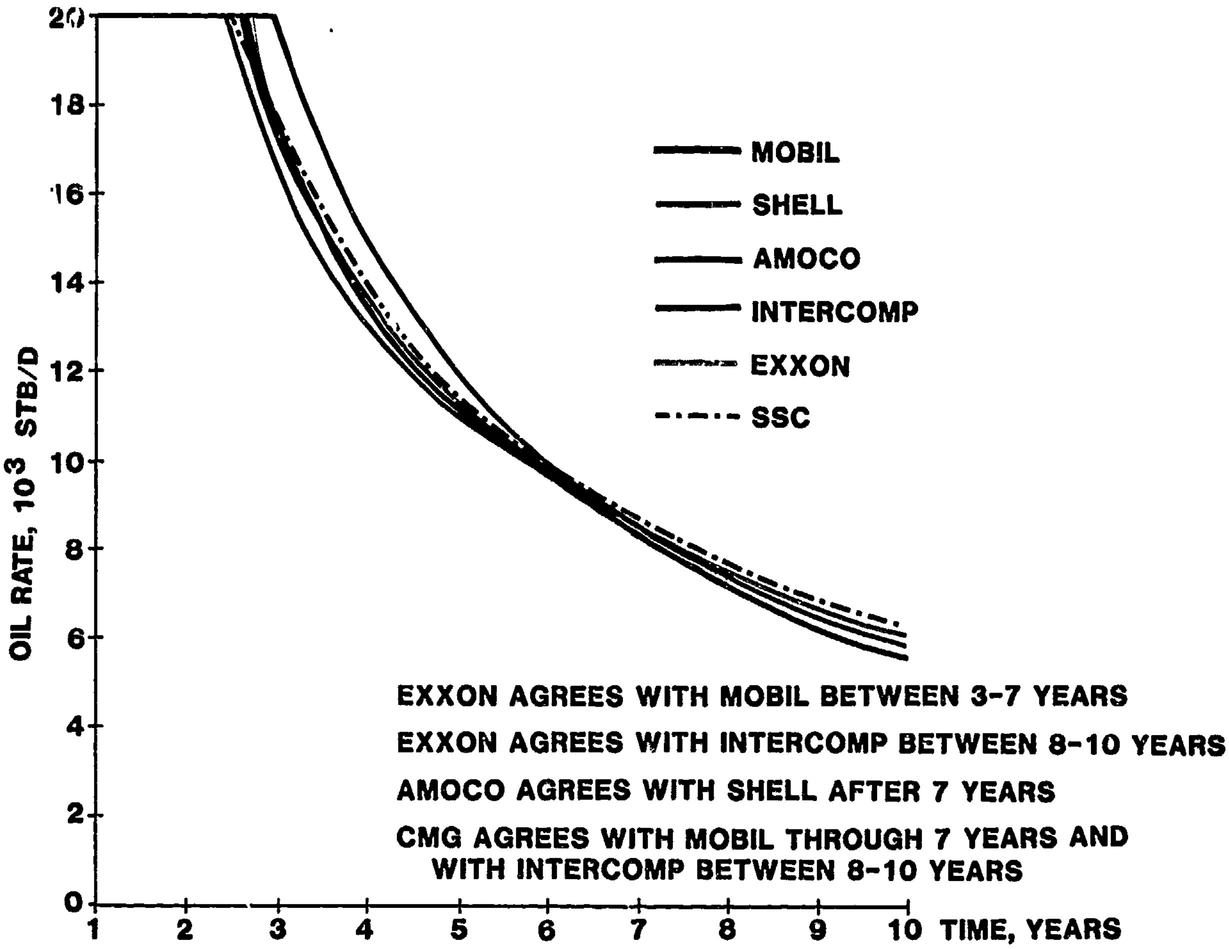

Fig. 3 – Case 1 – oil rate vs. time.

Results

A comparison of the results is given in Figs. 3 through 18. No comparison of saturation pressures is given because the values reported by the seven companies were within 20 psi of each other.

Description of the Simulators

Amoco's Model

The IMPES method was used, with semi-implicit adjustments in well rates. This method proved quite satisfactory; additional computations for implicit handling of interblock flow were not needed.

Maximum time-step size can vary with time and is input. The model determines internal time-step sizes to satisfy both the current maximum Δt and the maximum saturation change for any grid block (5% PV). A sequence of runs using maximum Δt of 0.25, 0.5, 1, and 2 months yielded virtually identical results, confirming the applicability of the IMPES method. The final results are for a maximum Δt of 1.0 month.

For each internal time step the computation sequence was as follows.

1. Well rates.
2. Coefficients including terms for semi-implicit production rates.
3. Iterative computation of grid-block pressure

TABLE 3 – RELATIVE PERMEABILITY DATA*

	Oil-Gas	
S_g	k_{rg}	k_{ro}
0	0.0	1.0
0.001	0.0	1.0
0.02	0.0	0.997
0.05	0.005	0.980
0.12	0.025	0.700
0.2	0.075	0.350
0.25	0.125	0.200
0.3	0.190	0.090
0.4	0.410	0.021
0.45	0.60	0.010
0.5	0.72	0.001
0.6	0.87	0.0001
0.7	0.94	0.000
0.85	0.98	0.000
1.0	1.0	0.000

*This is a two-phase, gas/oil problem. Set the relative permeability to water equal to zero for all values of water saturations.

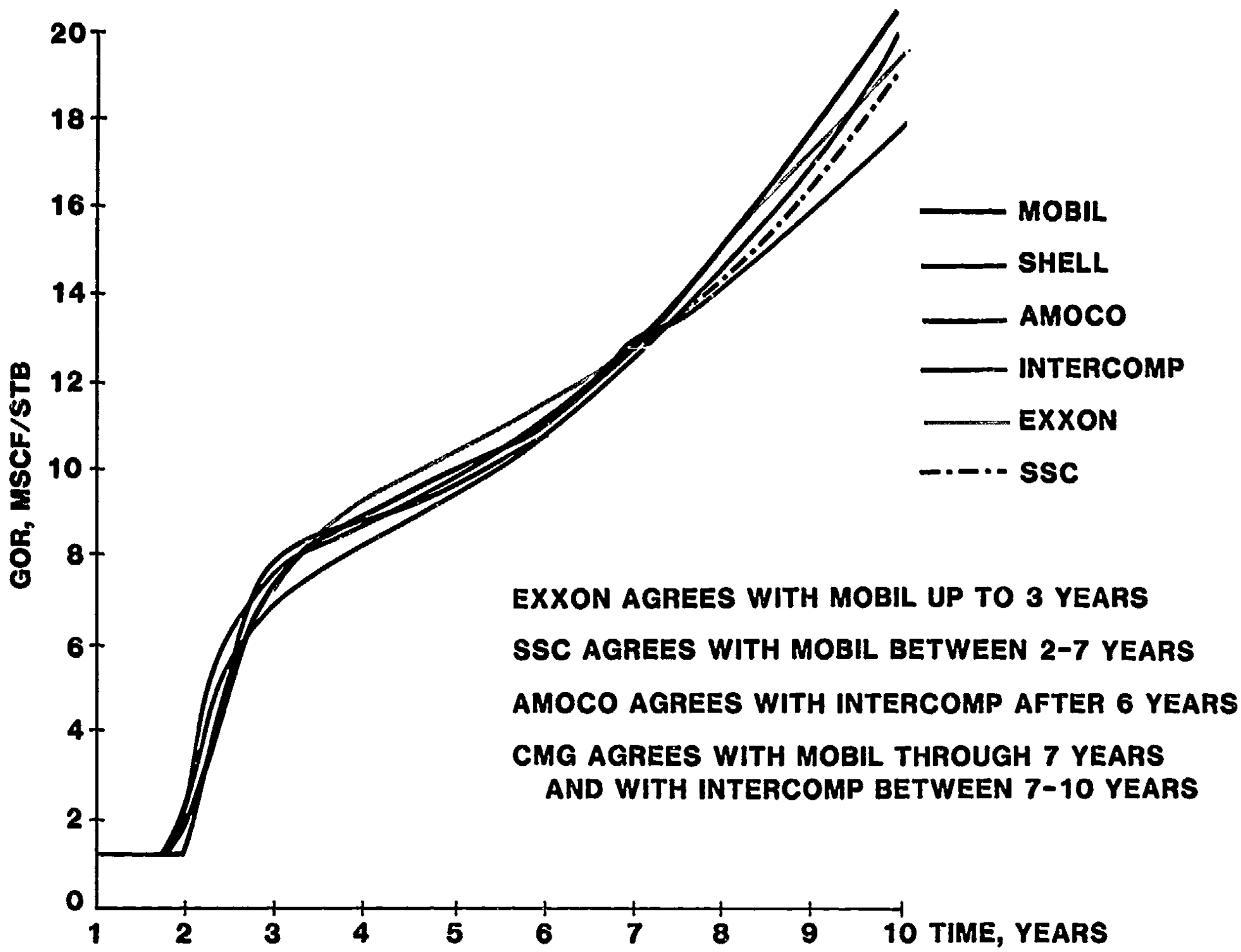

Fig. 4 – Case 1 – GOR vs. time.

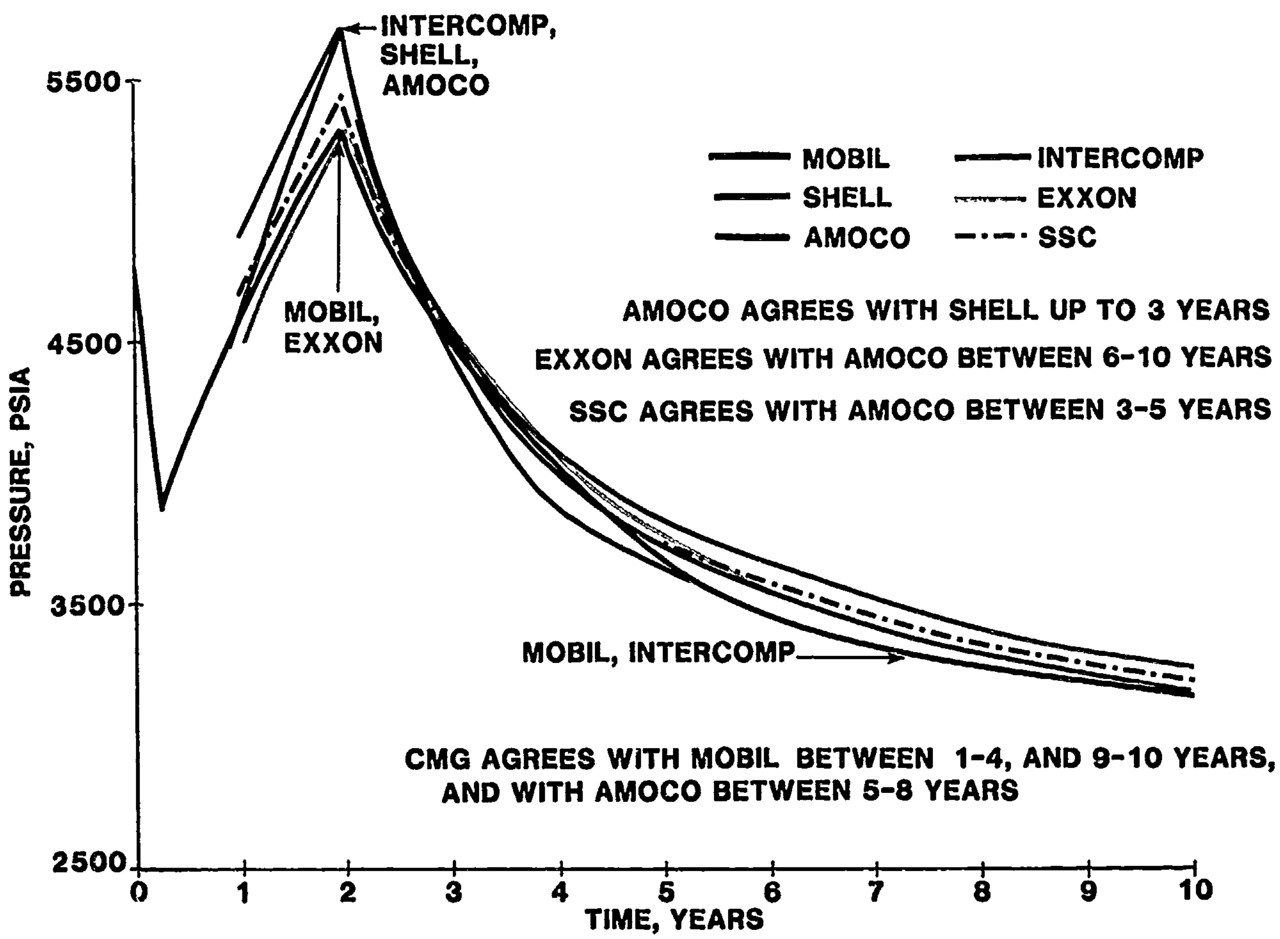

Fig. 5 – Case 1 – pressure vs. time for producing well Cell 10, 10, 3.

changes using slice successive overrelaxation.

4. Noniterative computation of grid-block saturation changes and of semi-implicit adjustments in production rates.

5. If any saturation change exceeds the maximum, reduce Δt and go to Step 2.

6. Noniterative computation of grid-block bubble-point pressure changes (for variable saturation-pressure case only).

Production rates for each step were the sum of rates at the start of the step plus semi-implicit adjustments as saturations changed in the well block. An important exception is that the oil rate was held constant if, at the start of the step, the well had excess computed productivity (i.e., if the computed bottomhole pressure exceeded the minimum value of 1,000 psi).

CMG's Black-Oil Model

CMG's black-oil simulator models three-phase water/oil/gas systems or two-phase water/oil systems. The model includes the effects of gravity and capillary pressure. It can be run in the one-, two- or three-dimensional mode. Variable grid spacing can be used. The nonlinear equations are solved by Newtonian iteration with the derivatives of the Jacobian matrix evaluated numerically. The model contains several possible options for the weighting of mobilities. These include single-point upstream, two-point upstream, and centralized upstream weightings. The time discretization is by backward differences with a modified Crank-Nicholson method included as an option. The well model permits the placing of wells at various positions in a grid block. Multiblock completion wells are included and are modeled in a manner which does not increase the matrix bandwidth. Finally, an efficient solution routine is included in the model. This routine provides Gaussian elimination with block D4 ordering, a bandwidth-reducing option, and two different iterative solutions methods: AB and COMBINATIVE.[1]

The model is fully implicit in its basic formulation. It becomes highly implicit, not fully implicit, when the options for two-point upstream or centralized upstream weightings are used or when multiblock completion wells are modeled.

Disappearance of the gas phase is not handled by the conventional variable substitution technique but by a novel pseudo solution-gas formulation.[2] The pseudo solution-gas formulation allows both variable bubble-point problems and fixed bubble-point problems to be handled in a simple manner.

For this problem the simulator was run in three-phase, three-dimensional mode. The basic fully implicit formulation was used. The time discretization was backward differences. The matrix problem was solved by the AB iterative routine.[1]

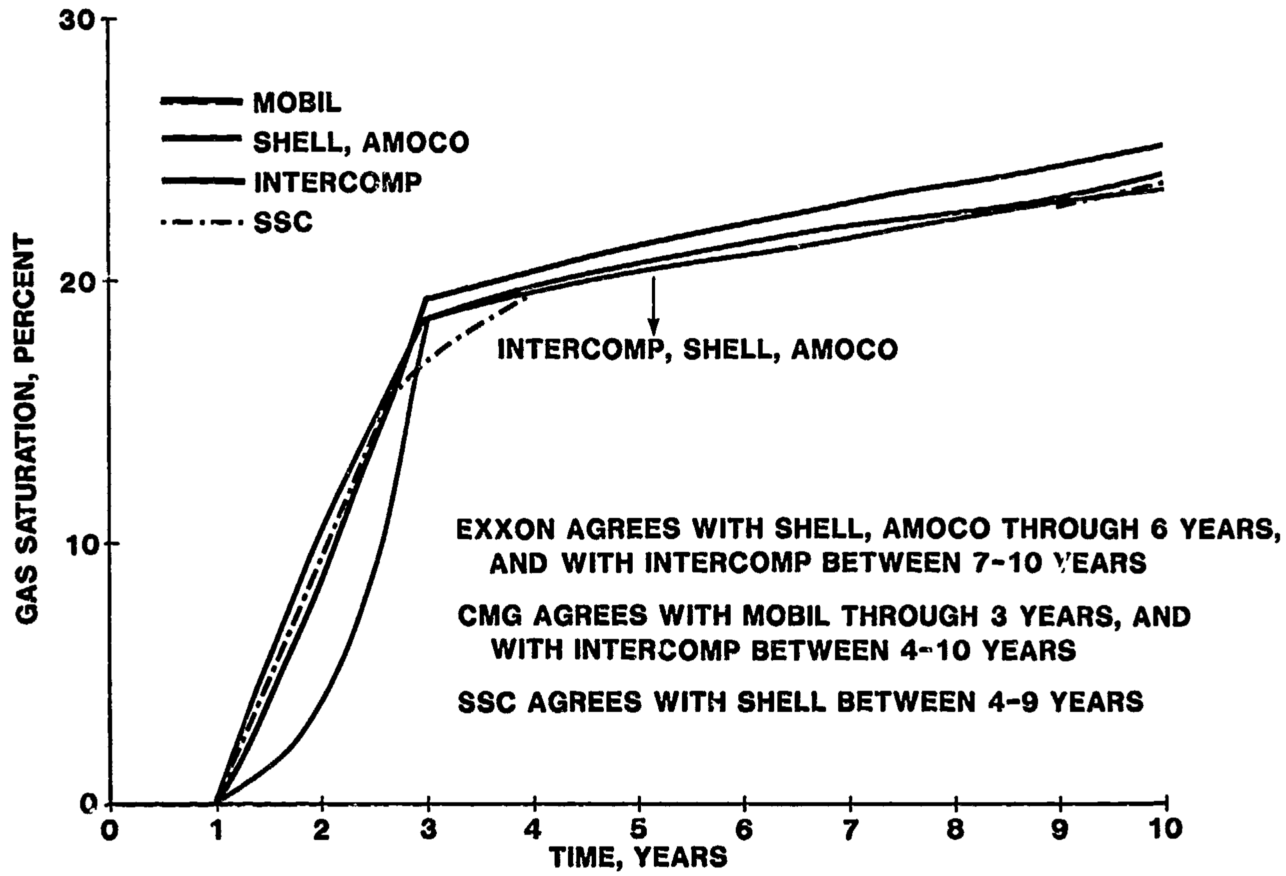

Fig. 6 – Case 1 – gas saturation vs. time for producing well Cell 10, 10, 3.

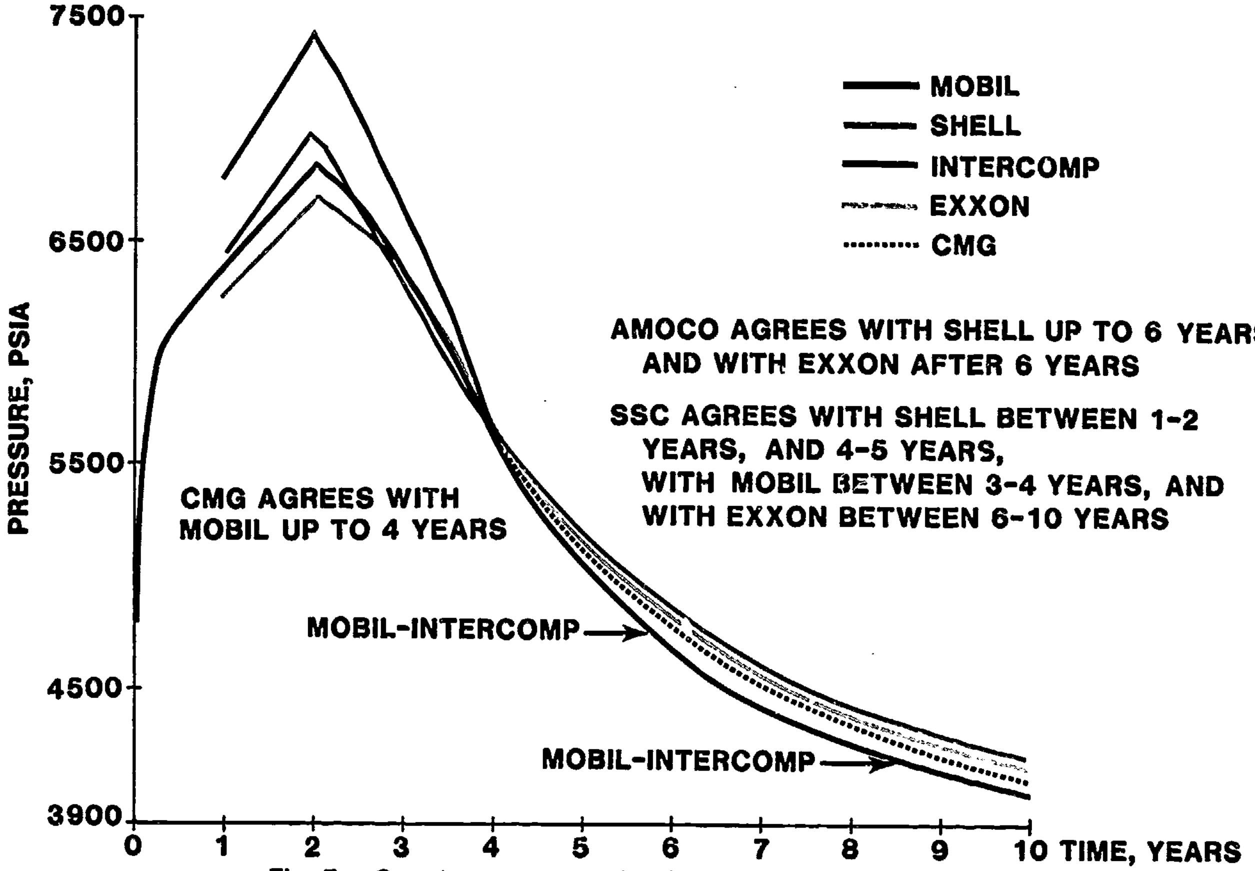

Fig. 7 – Case 1 – pressure vs. time for injection well Cell 1, 1, 1.

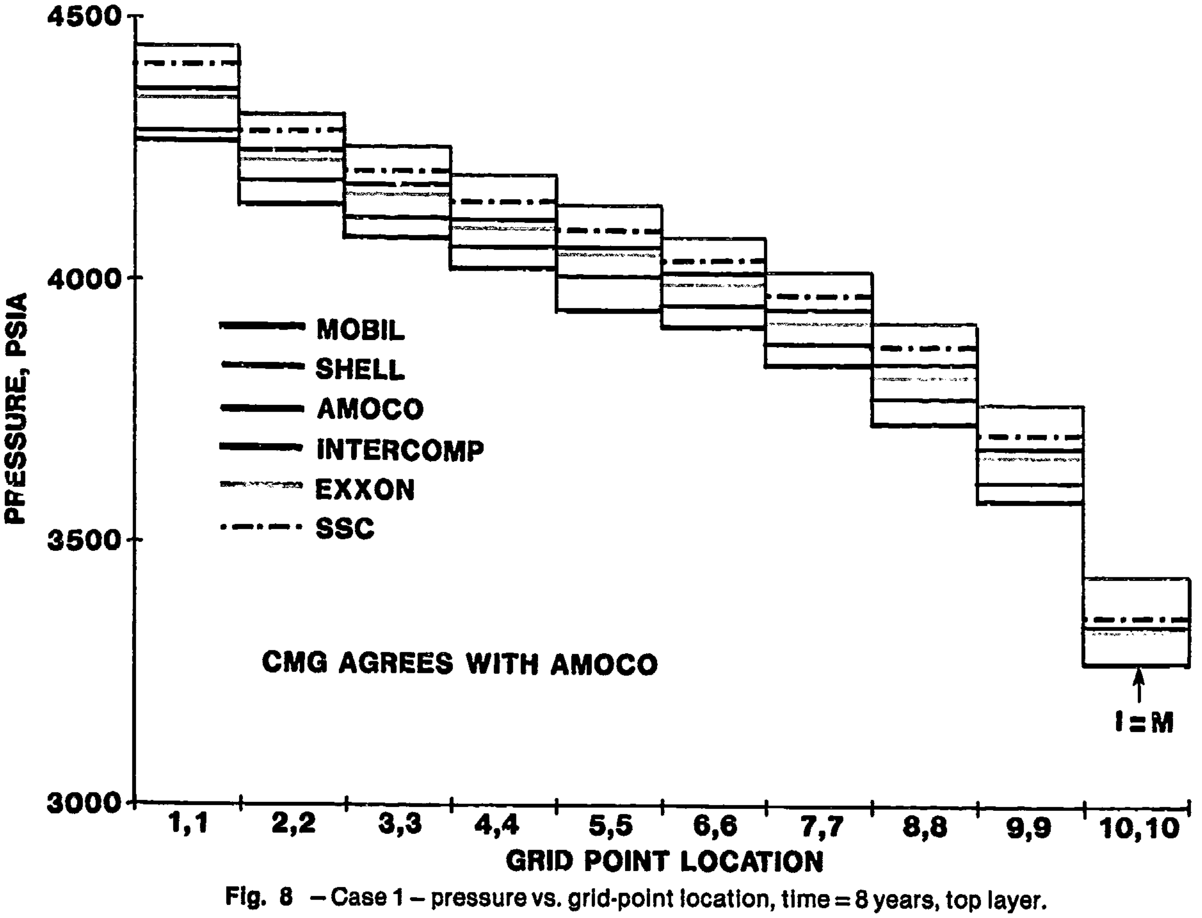

Fig. 8 – Case 1 – pressure vs. grid-point location, time = 8 years, top layer.

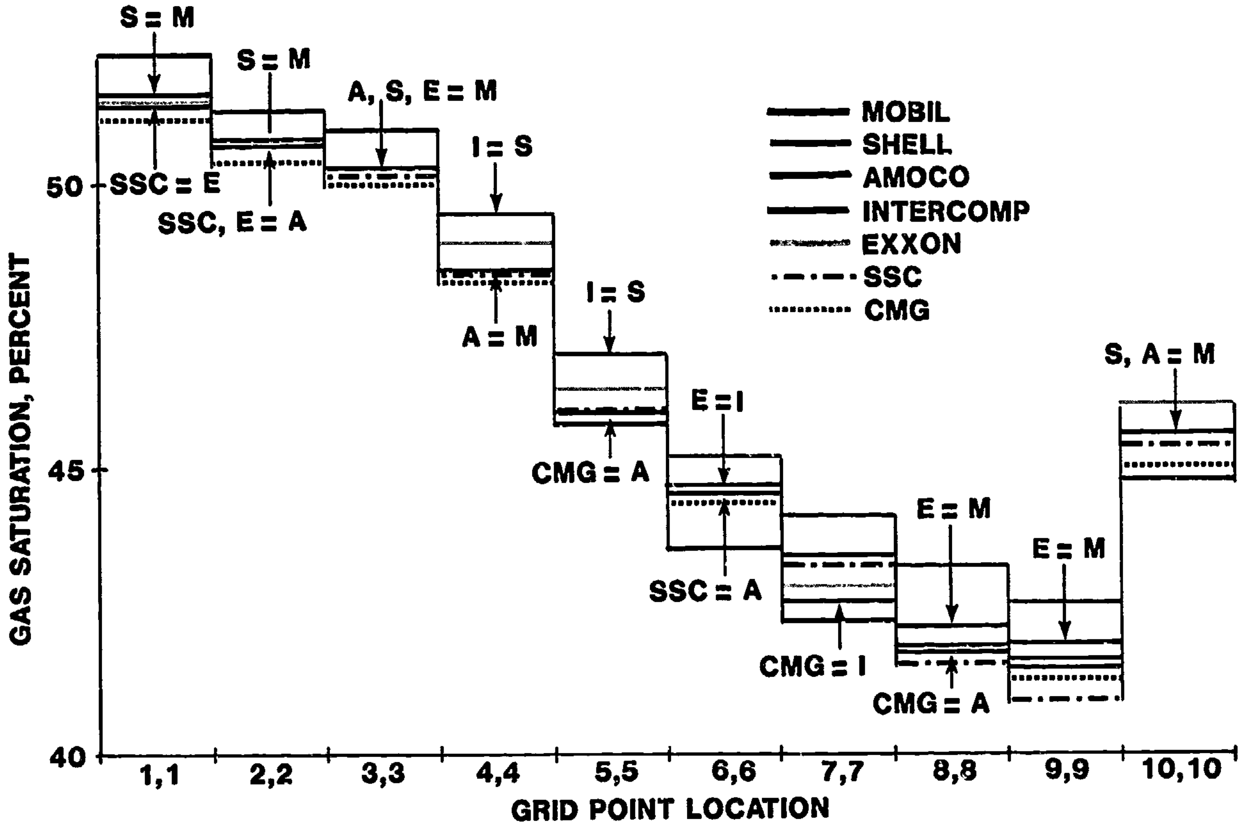

Fig. 9 – Case 1 – gas saturation vs. grid-point location, time = 8 years, top layer.

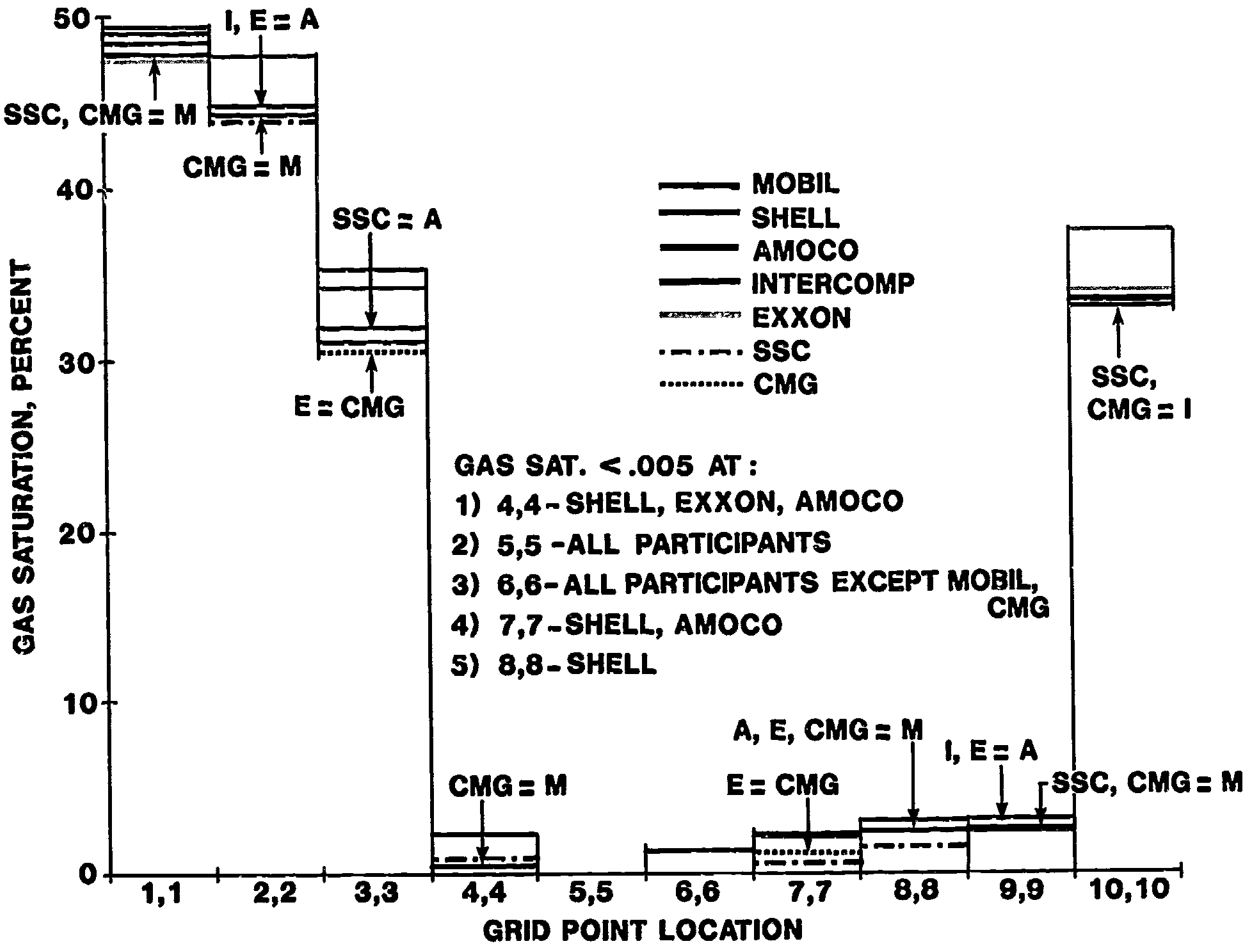

Fig. 10 – Case 1 – gas saturation vs. grid-point location, time = 8 years, middle layer.

Exxon's General Purpose Simulator

Exxon's general purpose reservoir simulator (GPSIM) uses a sequential implicit solution procedure.[3] The first step in this approach is the solution of a set of pressure equations. This set consists of a single equation for each grid block, and solving it yields a complete new pressure distribution at the end of a time step.

This pressure distribution then is used to calculate the sum of the velocities of all phases at each boundary between grid blocks, and these total velocities are used in a set of saturation equations. If either capillary pressure or relative permeability is being treated semi-implicitly, this set consists of two coupled equations per grid block and is solved simultaneously to yield saturation distributions at the new time. Otherwise, the equations are uncoupled and can be solved point by point explicitly, in the normal IMPES fashion.

Several options are available for solving the matrices involved. In the problem discussed here, a preconditioned conjugate gradient method[4] was used to solve for pressures, and strongly implicit procedure (SIP) was used to solve for saturations. (The full saturation solution was needed because mobilities were treated semi-implicitly.)

As is common in modern reservoir simulators, GPSIM can account for reservoir heterogeneity, rock compressibility, and solution of gas in both oil and water. Less common features it can model are vaporization of oil into the gas phase and hysteresis in the capillary pressure and relative permeability data.

GPSIM has only minor restrictions on the number of grid blocks that it can use; large problems can be run using only relatively modest amounts of central memory. This desirable feature is accomplished by using disks to store data temporarily by planes for three-dimensional problems or by rows for two-dimensional ones. If the central memory made available is sufficiently large, the program automatically will eliminate the temporary data storage, keeping all data within core.

Intercomp's Black-Oil Simulator

Intercomp's BETA II black-oil model is designed to simulate numerically two- or three-phase compressible flow in heterogeneous hydrocarbon reservoirs. Gas is assumed to be soluble in oil but not in water; neither oil nor water can exist in any phase other than its own. Solutions are obtained in one, two, or three spatial dimensions using either rectangular or cylindrical coordinates. Alternate solution procedures provide for efficient modeling of all classes of black-oil reservoir problems, ranging from individual well behavior (coning simulations or well test analysis) to large, multireservoir fields. To complement the three-phase simulation capabilities, BETA II contains two distinct segments of code

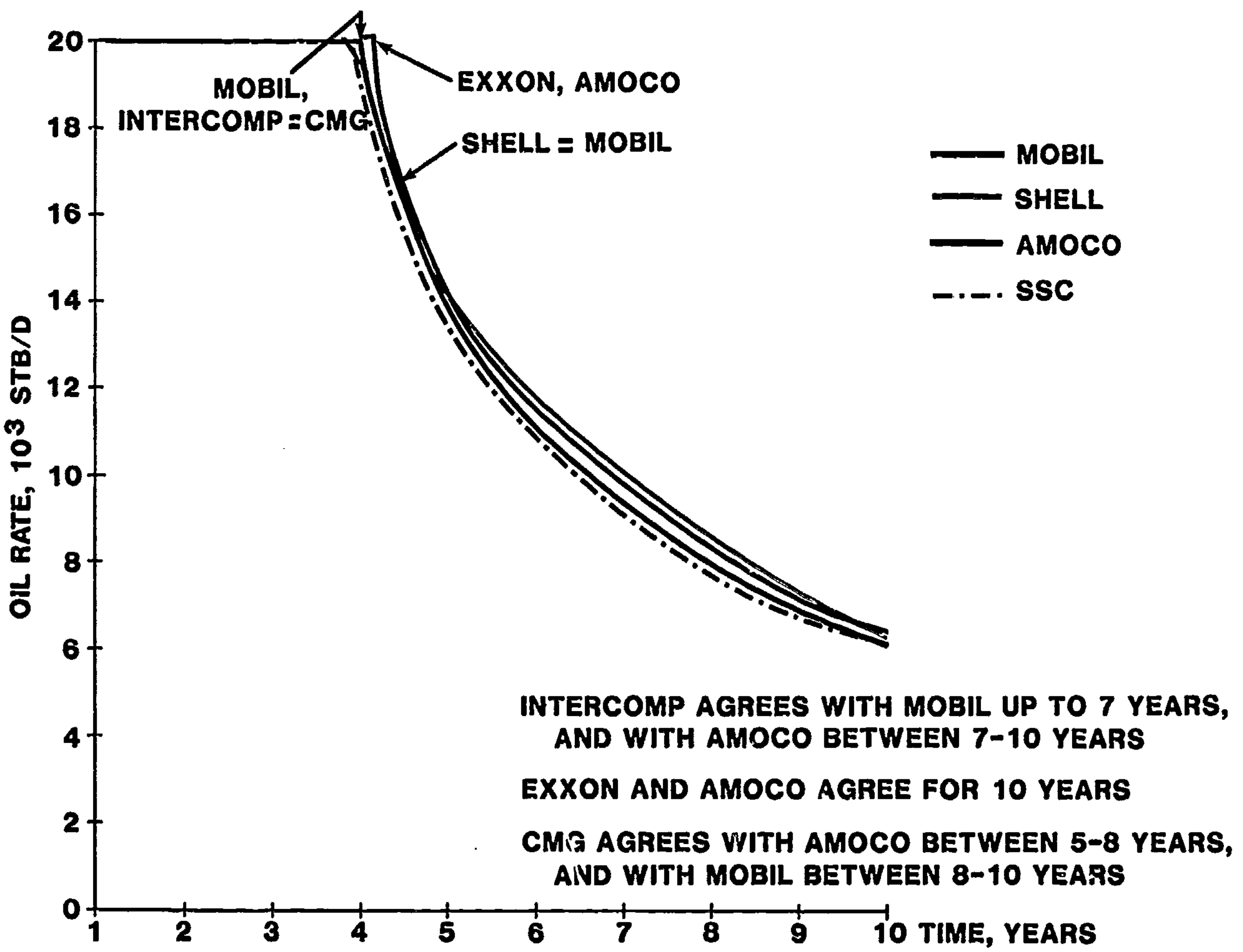

Fig. 11 – Case 2 – oil rate vs. time.

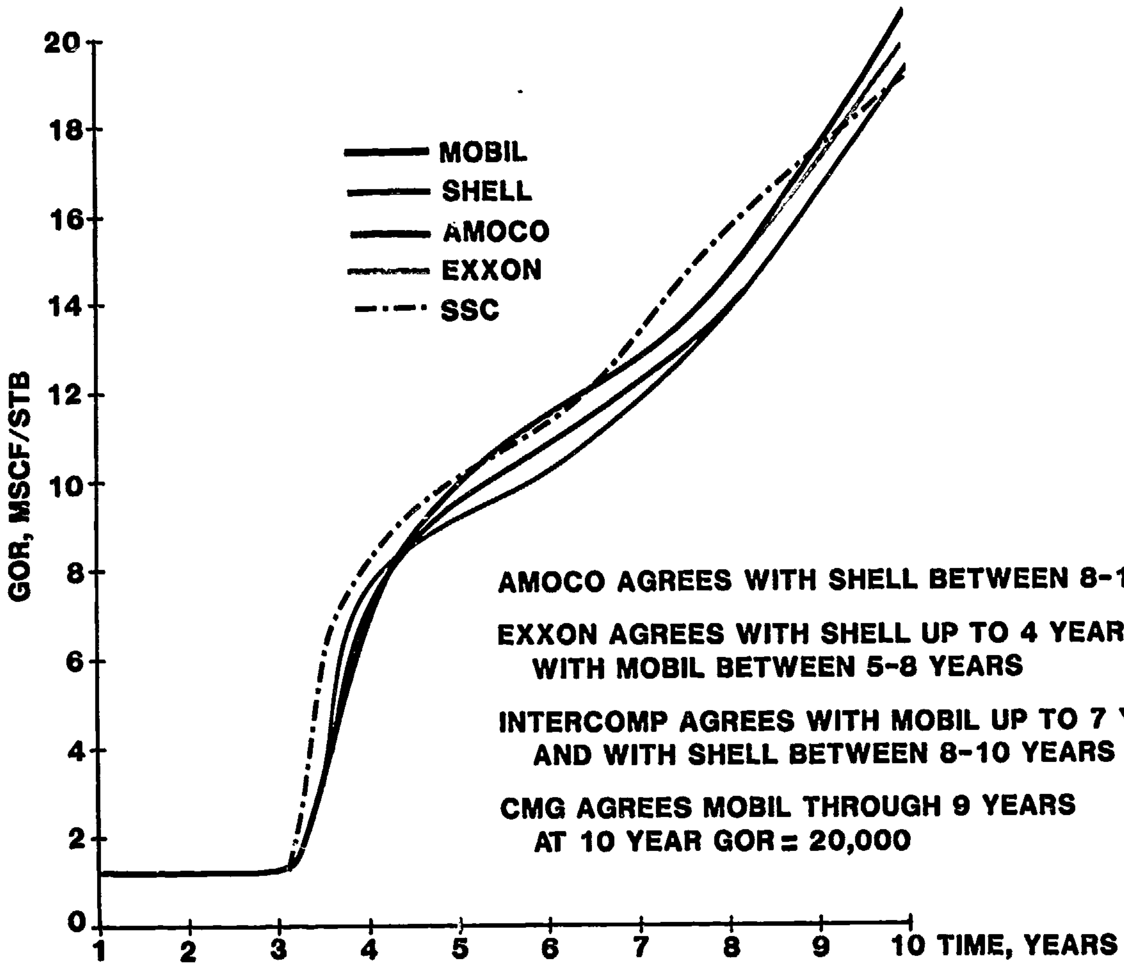

Fig. 12 – Case 2 – GOR vs. time.

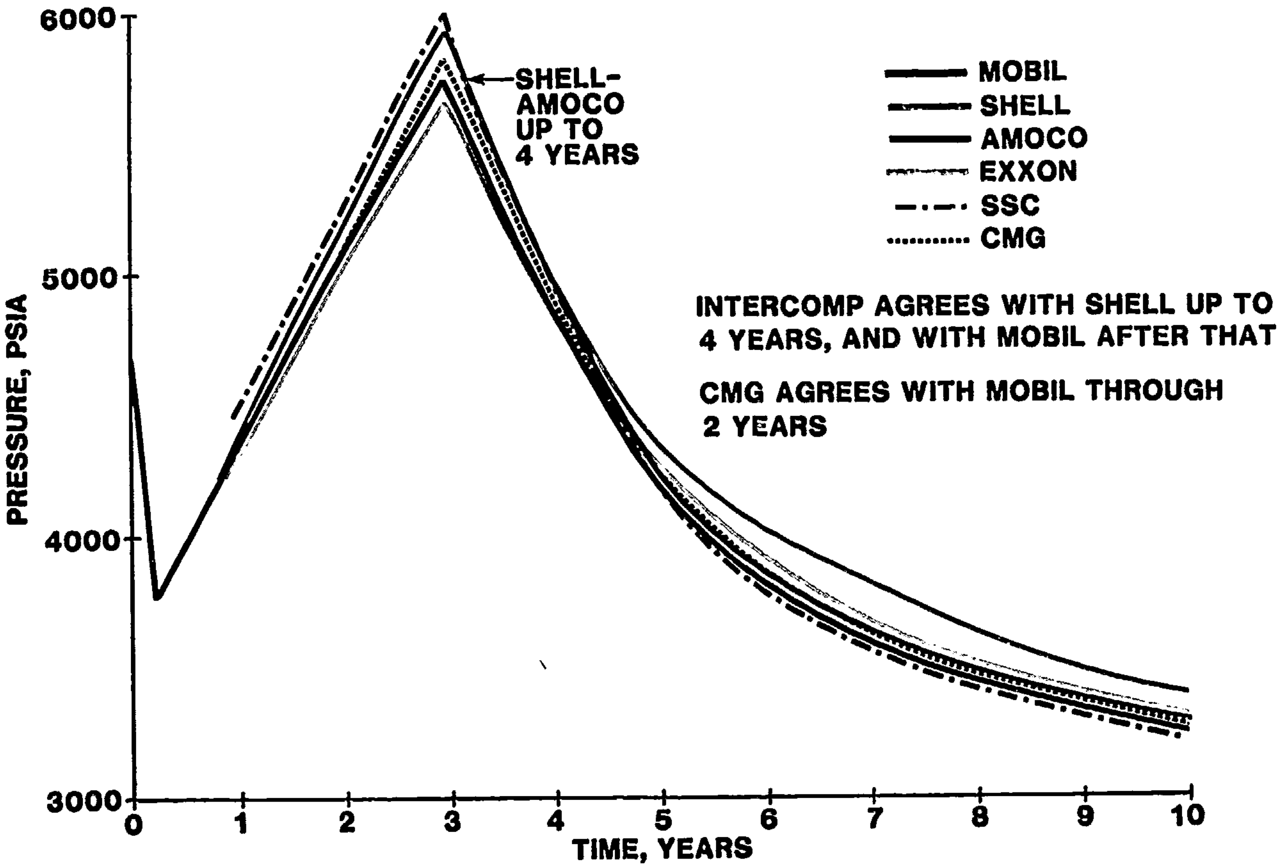

Fig. 13 – Case 2 – pressure vs. time for producing well Cell 10, 10, 3.

which are designed specifically for optimal solution of two-phase problems: one for water/oil (dissolved-gas content constant) problems and one for gas/water (no oil phase) problems. BETA II has a large variety of user-oriented features such as input/output options and well control options.

There is a one-equation implicit pressure formulation in which the equations are decoupled and solved in this order: pressure, gas saturation, and water saturation. For the one-equation formulation, there are options to solve for both saturations explicitly (IMPES) and either or both saturations implicitly (sequential). There is an alternative two-equation formulation in which the program solves implicitly for pressure and gas saturation and then solves for water saturation. Similar to the previous formulation, water saturation can be treated either implicitly or explicitly. In any of the formulations, multiple outer iterations may be taken to account for the nonlinearity of the basic flow equations. However, if no vaporization or resolution of gas is occurring, only one or two iterations are required to converge the nonlinearities adequately.

The large systems of linear algebraic equations may be solved by a variety of methods, any one of which may offer significant speed advantages on a given problem: (1) direct solution by reduced band-width Gaussian elimination, (2) several forms of one-line, two-line, and planar successive overrelaxation (SOR), or (3) SIP.

Mobil's All Purpose Reservoir Simulator (ALPURS)

ALPURS is a three-dimensional, three-phase, multiwell, black-oil reservoir simulator which uses a strongly coupled, fully implicit method to solve simultaneously for all unknowns.[5] The nonlinear intercell flow equations and well-constraint equations are linearized and iterated to converge using Newton-Raphson iteration. Linear equations are solved with block successive overrelaxation. A typical block is an *x-z*, *y-z*, or *x-y* reservoir slice, which is solved by sparse elimination. The relaxation parameter is computed automatically using the power method and Rayleigh quotients. ALPURS accounts for reservoir heterogeneity, rock compressibility, gravity, gas dissolved in both the oil and water phases, constant or variable bubble-point pressures, hysteresis in saturation-dependent data, tubing string pressure drop, and flash surface separation. Modern concepts of well flow equations are incorporated, including pseudo gas-potential function, skin factor to account for damage or improvement, non-Darcy flow effect, and flow restriction due to limited entry such as partial penetration.

SSC Model

SSC's black-oil model employs an Adaptive Implicit Method (AIM). This technique, which recently was developed at SSC, seeks to achieve an optimum with

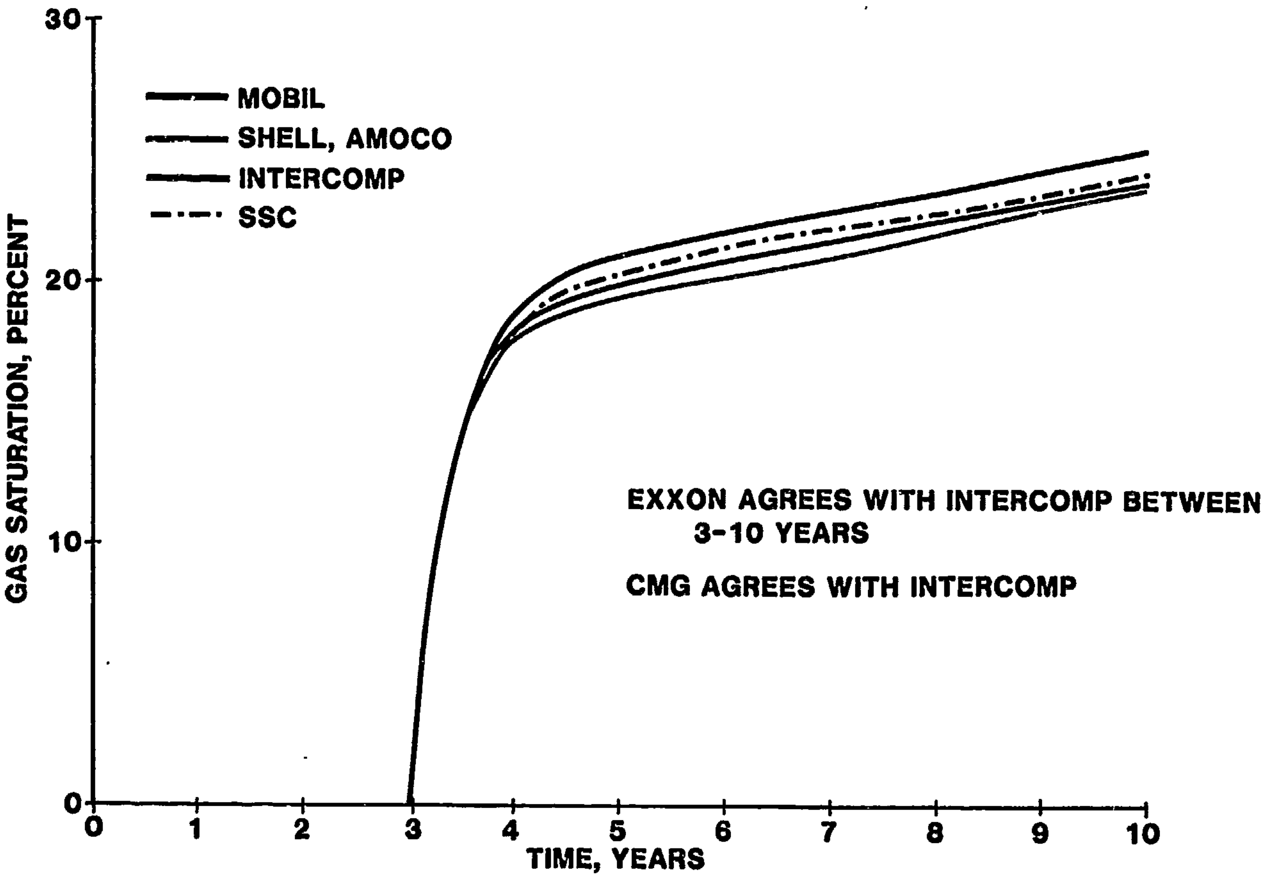

Fig. 14 – Case 2 – gas saturation vs. time for producing well Cell 10, 10, 3.

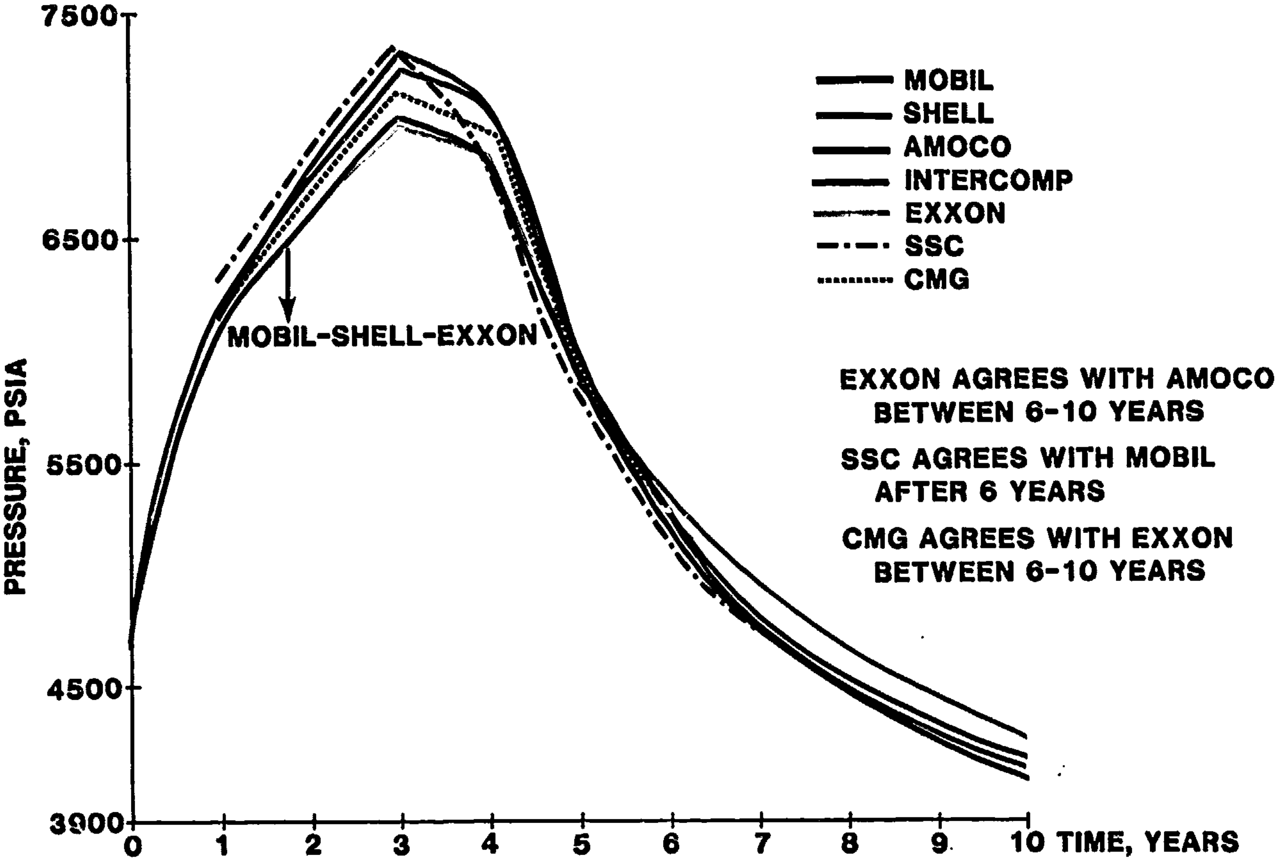

Fig. 15 – Case 2 – pressure vs. time for injection well Cell 1, 1, 1.

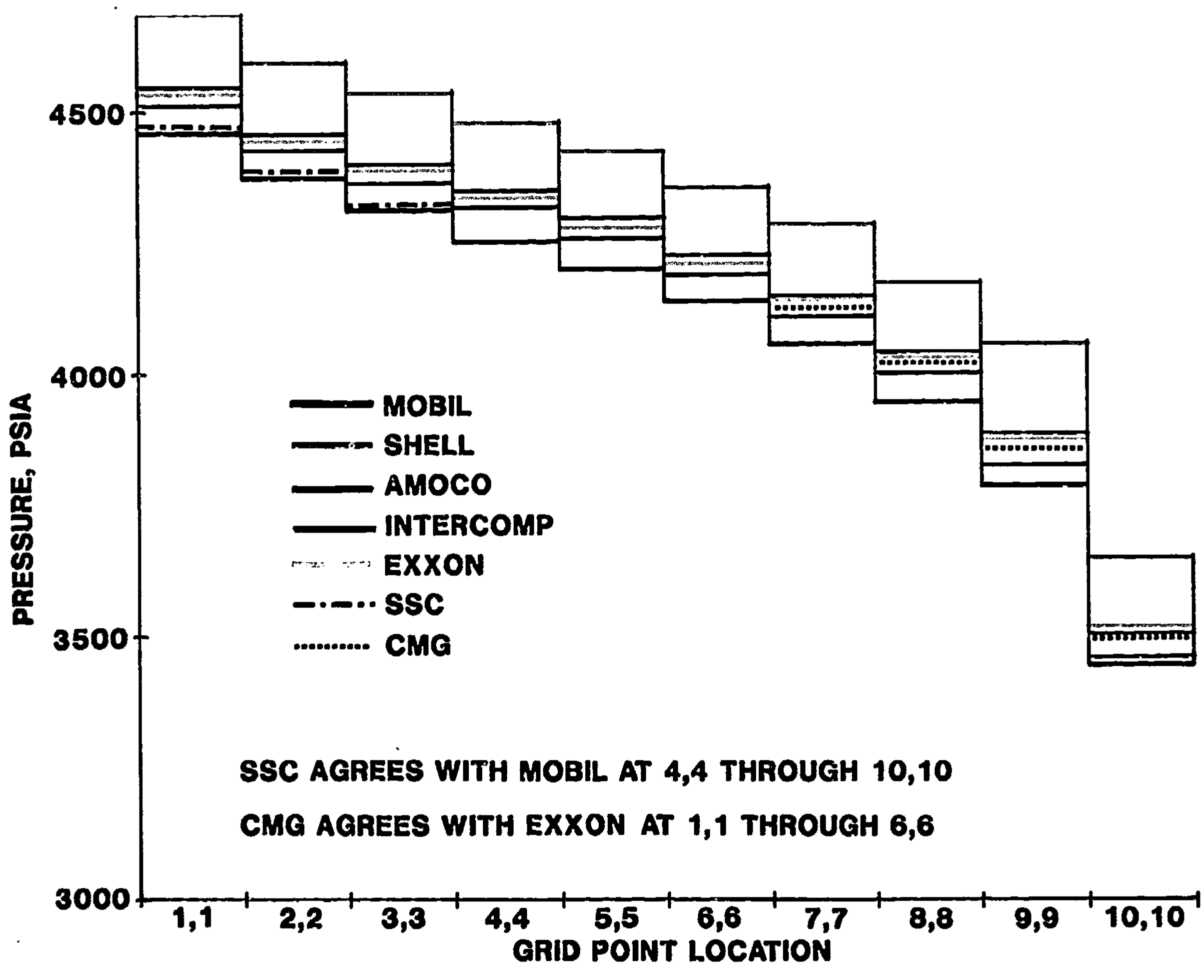

Fig. 16 – Case 2 – pressure vs. grid-point location, time = 8 years, top layer.

respect to stability, truncation errors, and computer costs. Typically, only a small fraction of the total number of grid blocks during simulation experience sufficiently large surges in pressure and/or saturation to justify implicit treatment. When it is needed, implicit treatment may not be required in all phases or for long periods of time. Moreover, those cells requiring implicit treatment will change as the simulation proceeds. Consequently, a model offering a fixed degree of implicitness to *all* cells for *all* time steps is not always the most desirable. For example, a fully implicit model, while ensuring stable answers, amounts to overkill in most of the cells most of the time, while an IMPES model can cause underkill. With AIM there is no problem of over- or underkill. Various degrees of implicitness are invoked regionally or individually cell by cell – i.e. the solution is advanced with adjacent cells having different degrees of implicitness. As the calculations proceed, the degrees of implicitness locally and dynamically shift as needed – all automatically. The whole idea is to apply the right amount of implicitness where and when needed and for only as long as needed.

The simulator also provides a wide variety of user-oriented features. For example, one can override AIM and operate in a fully implicit, partially implicit, or an IMPES mode. Variable bubble-point problems, such as that in Case 2, are handled by variable substitution. The simulator is a general purpose package offering three-dimensional capability in both Cartesian and cylindrical coordinates.

Shell Development Model

The Shell reservoir simulation system operates an IMPES mode or an implicit mode. There are three pseudocomponents: water, stock-tank oil, and separator gas. There are three phases: aqueous, hydrocarbon liquid, and hydrocarbon vapor. The aqueous phase contains water. The hydrocarbon liquid and vapor phases can contain both oil and gas. A fourth component is also available for modeling polymer or carbon dioxide. There are several indirect and direct solution methods as a user option. Additionally, two-point upstream weighting is used to calculate phase mobilities.

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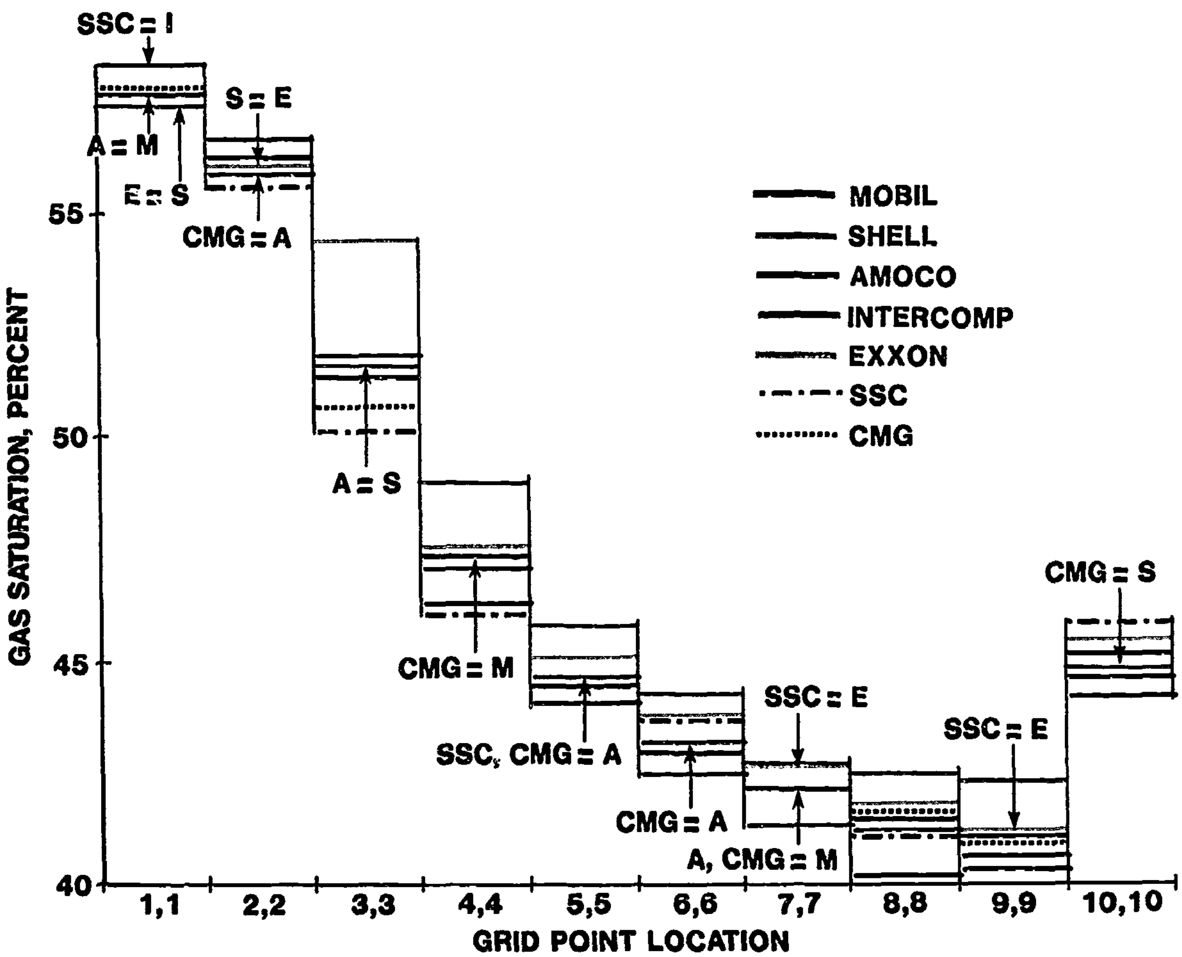

Fig. 17 – Case 2 – gas saturation vs. grid-point location, time = 8 years, top layer.

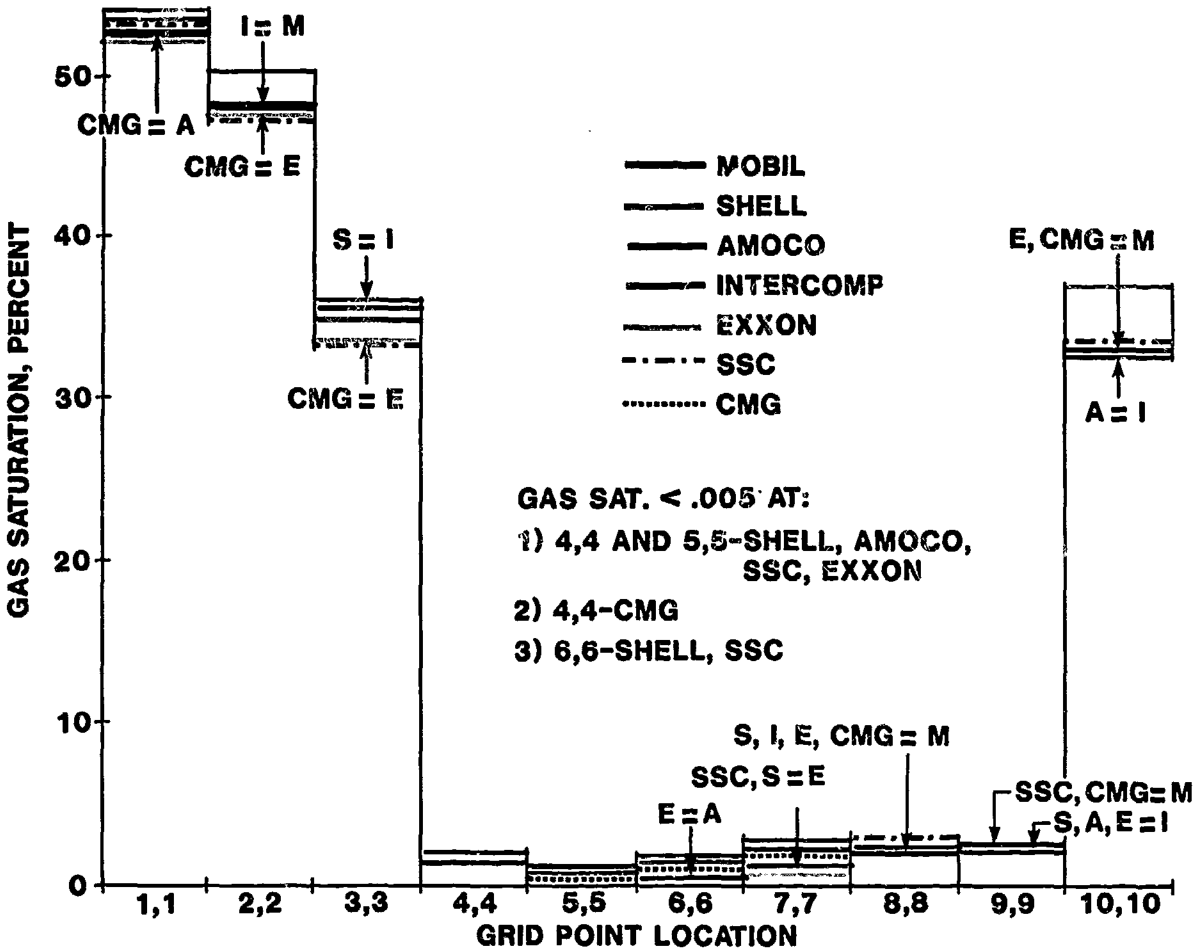

Fig. 18 – Case 2 – gas saturation vs. grid-point location, time = 8 years, middle layer.

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Nomenclature

h = thickness
I = number of grid points in the x direction
J = number of grid points in the y direction
k = permeability
k_{rg} = relative permeability to gas
k_{ro} = relative permeability to oil
k_x = permeability in the x direction
k_y = permeability in the y direction
$M(p)$ = pseudo gas potential
S_g = gas saturation
S_o = oil saturation
S_w = water saturation
Δt = time step
ϕ = porosity

SI Metric Conversion Factors

bbl	×	1.589 873	E−01	=	m^3
cp	×	1.0*	E−03	=	Pa·s
cu ft	×	2.831 685	E−02	=	m^3
°F		(°F−32)/1.8		=	°C
ft	×	3.048*	E−01	=	m
lbm	×	4.535 924	E−01	=	kg
psi, psia	×	6.894 757	E+00	=	kPa
scf	×	2.863 640	E−02	=	std m^3

*Conversion factor is exact.

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